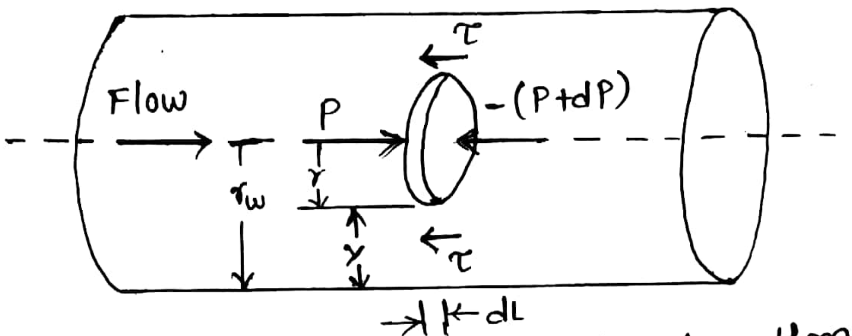
* Flow of incompressible Fluids in pipes:-

If the fluid density is an appreciably affected by ranges in temperature and pressure, the fluid is said to be compressible fluid.

* Shear stress distribution in pipes:-



Fluid element in steady flow through pipe

A fluid of constant density is flowing steady through a horizontal pipe. consider a disk shaped element of fluid, shown in fig. having radius 'r' & length 'dL' concentric with the axis of the pipe.

The fluid pressure on the upstream face, of the disk is $P = P_a$ &

The fluid pressure on the downstream face of the disk is $P + dP = P_b$

wall force $= 0$

gravity force $= 0$

Momentum force, $\dot{m}v - \dot{m}v = 0$

The momentum equation is applied between the two faces of the disk. Since the flow is fully developed,

$$S_a = S_b = \pi r^2$$

where S_a & S_b = cross-sectional area at locations a & b respectively $S_a = S_b = \pi r^2$, $P_a = P$, $P_a S_a = \pi r^2 P$, $P_b S_b = (\pi r^2)(P+dP)$

The shear force (F_s) acting on the rim of the element is the product of the shear stress (τ) and cylindrical area $[2\pi r dL]$

$$F_s = 2\pi r dL \tau$$

where r = radius of the cylinder (m)

$$F_s = \pi r^2 P - \pi r^2 (P+dP) - 2\pi r dL \tau = 0$$

$$\pi r^2 dP + 2\pi r dL \tau = 0$$

Dividing by $\pi r^2 dL$

$$\frac{dP}{dL} + 2\frac{\tau}{r} = 0 \quad \text{------------ (1)}$$

$\pi r^2 P - \pi r^2 P - \pi r^2 dP - 2\pi r dL \tau$

$$\frac{\pi r^2 dP + 2\pi r^2 dL \tau}{\pi r^2 dL} = 0$$

$$\frac{dP}{dL} + 2\frac{\tau}{r} = 0$$

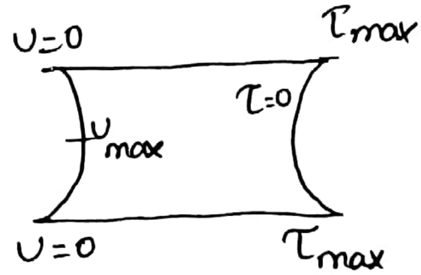
In steady flow, either laminar (or) turbulent flow the pressure at any given cross-section of a pipe is constant.

Hence, $\left(\frac{dP}{dL}\right)$ is independent of r

The above equation can be written for the entire cross-section of the pipe with

$\tau = \tau_w$ &

$r = r_w$



where, τ_w = shear stress at the wall

r_w = radius of the pipe wall

at wall,

$$\frac{dP}{dL} + 2\frac{\tau_w}{r_w} = 0 \qquad \text{——— (2)}$$

Substracting $\frac{dP}{dL} + 2\frac{\tau}{r} = 0$ from the above equation we get

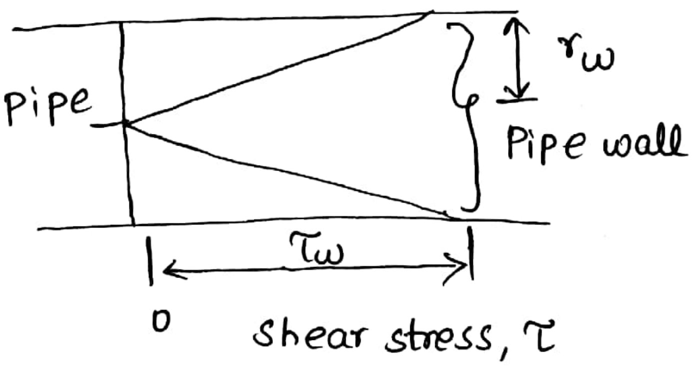
$$\frac{dP}{dL} + 2\frac{\tau_w}{r_w} - \frac{dP}{dL} - 2\frac{\tau}{r} = 0$$

$$2\frac{\tau_w}{r_w} - 2\frac{\tau}{r} = 0$$

$$2\frac{\tau_w}{r_w} = 2\frac{\tau}{r}$$

$$\boxed{\tau = \frac{\tau_w r}{r_w}} \qquad \text{——— (3)}$$

The above equation relates the shear stress with radous of pipe show in fig.



variation of shear stress in pipe

If $r = 0$

then $\tau = 0$

This linear relationship applies in both laminar & turbulent flow & to both Newtonian & Non-Newtonian flows.

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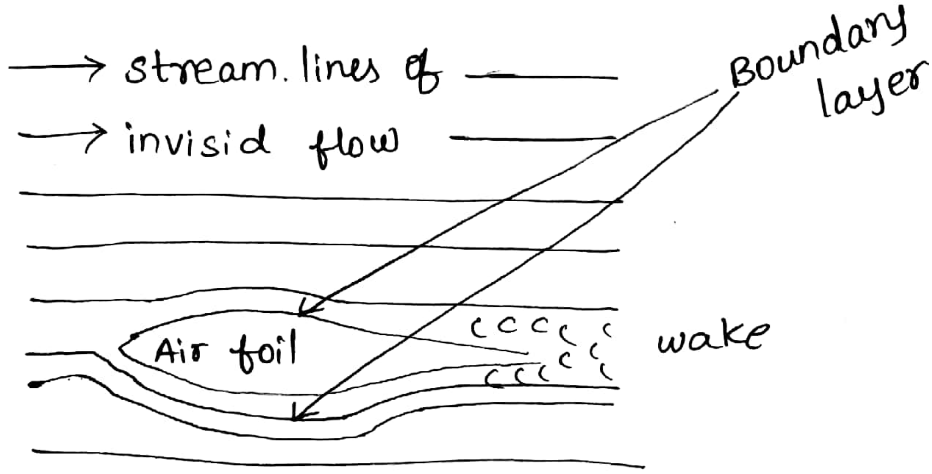
* Relation between skin friction & wall shear :-

Skin friction :-

Skin friction arises from the friction of the fluid against the "skin" of the object that is moving it and forms a vector at each point on the surface.

Ex:- Aerospace

Air foil → Minimum pressure occurs near leading edge increasing pressure distribution long regions of turbulent flow (high skin frim drag)



Friction drag also known as skin friction drag, is drag caused by the friction of a fluid against the surface of an object that is moving through it.

The boundary layer normally generates a drag on the plate as a result of the viscous stresses which are developed at the wall. This drag is normally referred to as skin friction.

Interaction between fluid and skin of the body.

wall shear :-

$$\boxed{\tau = \frac{F}{A}}$$

'wall shear' stress is the ~~is~~ ~~tt~~ shear stress in the layer of fluid next to the wall of the pipe.

→ The no. slip condition dictates that the speed of the fluid at the boundary is zero, but at some height from the boundary the flow speed must equal that of a fluid. The region between two points is called boundary layer.

→ The shear stress is imparted on to the boundary as a result of this loss of velocity.

* Relation for skin friction & wall shear:-

In a pipe flow

$$P_b - P_a = \text{pressure drop} = -\Delta P_s$$

(or)

$$\Delta P_s = P_a - P_b$$

$Z_a = Z_b$ [height 'Za' from ground level (or) datum $\begin{matrix} Z_a = 0 \\ Z_b = 0 \end{matrix}$]

$U_a = U_b$ [velocity throughout the pipe is const]

when Bernoullis equation is applied to pipe flow, only skin friction term exists & the pressure drop is due to the skin friction. Then

$$\frac{P_a}{\rho} + gz_a + \frac{U_a^2}{2} - h_f = \frac{P_b}{\rho} + gz_b + \frac{U_b^2}{2}$$

$$\frac{P_a}{\rho} = \frac{P_a}{\rho} - \frac{\Delta P_s}{\rho} + h_{fs}$$

Energy loss due to skin friction, $h_{fs} = \frac{\text{Pressure drop}}{\rho} = \frac{\Delta P_s}{\rho}$

For unit length, $h_{fs} = \frac{\Delta P_s}{\rho L}$ and

we have [from shear stress distribution equation]

$$\left.\begin{aligned} \frac{\Delta P_s}{L} &= \frac{2\tau_w}{r_w} \\ \frac{\Delta P_s}{\rho} &= h_{fs} \end{aligned}\right\}$$

$$\boxed{\Delta P_s = \frac{2\tau_w}{r_w} L}$$

$r = \frac{D}{2}$

$r_w = \frac{D}{2}$

$$\therefore \boxed{\frac{2\tau_w}{\rho r_w} L = h_{fs}}$$

$$\therefore h_{fs} = \frac{2\tau_w}{\rho r_w} L = \frac{4\tau_w}{\rho D} L$$

where, D is the diameter of the pipe

$$\therefore \boxed{h_{fs} = 4\frac{L}{D}\frac{\tau_w}{\rho}}$$

* **Fanning friction factor (f):-**

It is defined as the ratio of wall shear stress to the product of density & velocity head.

$$f = \frac{\tau_w}{\rho \frac{\bar{V}^2}{2}}$$

$$f = -\frac{dP}{2dL} \frac{r_w/\rho\bar{V}^2}{2}$$

$$f = -\frac{dP}{dL}\frac{r_w}{\rho\bar{V}^2}$$

$$f = \frac{\Delta P_s}{L} \cdot \frac{r_w}{\rho\bar{V}^2}$$

$$f = \frac{\Delta P_s}{L} \cdot \frac{D}{2\rho\bar{V}^2}$$

$$\boxed{\begin{aligned} \frac{2\tau_w}{r_w} &= \frac{-dP}{dL} \\ \tau_w &= \frac{-dP\, r_w}{2dL} \end{aligned}}$$

$$\boxed{h_{fs} = \frac{\Delta P_s}{\rho}}$$

$$\boxed{r_w = \frac{D}{2}}$$

$$\frac{\Delta P_s}{L} = \frac{2 f \rho \bar{V}^2}{D}$$

$$\therefore \quad h_{fs} = \frac{\Delta P_s}{\rho}$$

$$\boxed{h_{fs} = 4f \frac{L}{D} \frac{\bar{V}^2}{2}}$$

$$\boxed{\frac{\Delta P_s}{L} = \frac{2 f \rho \bar{V}^2}{D}}$$

$$\boxed{h_{fs} = 4 \frac{L}{D} \frac{\tau_w}{\rho}}$$

* **Hydraulic Radius (r_H) :-**

It is defined as the ratio of crosssectional area to the wetted perimeter.

$$r_H = \frac{S}{L_p}$$

[S - cross sectional area of channel
L_p - Perimeter of channel with fluid]

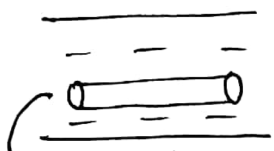
For a pipe

$$r_H = \frac{\pi/4 \, D^2}{\pi D}$$

$$\boxed{r_H = D/4}$$

$$D = 4 r_H$$

For annulous $D_{eq} = 4 r_H = D$



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$$N_{Re} = \frac{D_{eq} \bar{V} \rho}{\mu}$$

$$\boxed{r_H = \frac{D_o - D_i}{\mu}}$$

$$\boxed{4 r_H = D}$$

For square,

$$r_4 = \frac{s^2}{4s}$$

$$= \frac{s}{4}$$

$$\therefore \quad \boxed{s = 4 r_H}$$

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* Laminar Flow in pipes :-

When the fluids tend to flow without lateral mixing and adjacent layers slide past one another, then flow is called laminar flow.

In pipes because of symmetry about the axis the local velocity (u) depends on the radius (r) only.

The velocity distribution for fully developed flow of Newtonian fluid for laminar flow is given below.

In a circular ring of radius 'r' & width 'dr'

element cross-section = ds

$$ds = 2\pi r\, dr$$

From Newton's law of viscosity

$$\tau = -\mu \frac{du}{dr}$$

$$\frac{\tau}{r} = \frac{\tau_w}{r_w}$$

$$\boxed{\tau = \frac{\tau_w r}{r_w}}$$

$$\Rightarrow \mu = \frac{-\tau}{du/dr}$$

$$\frac{du}{dr} = -\frac{\tau}{\mu}$$

$$\boxed{\frac{du}{dr} = \frac{-\tau_w}{r_w}\frac{r}{\mu}}$$

$$\boxed{\frac{du}{dr} = -\frac{\tau_w}{r_w \mu} r} \qquad \text{——— (1)}$$

The minus sign in the equ(1) accounts for the fact that in a pipe u decreases as r increases.

Integrate equ(1) with the boundary condition $u=0, r=r_w$ gives

$$\int_0^u du = -\frac{\tau_w}{r_w \mu}\int_{r_w}^{r} r\,dr$$

$$\boxed{u = \frac{\tau_w}{2 r_w \mu}(r_w^2 - r^2)} \quad \text{——— (2)}$$

The maximum value of the local velocity is denoted by U_{max} and is located at the center of pipe. The value of U_{max} is found from equ(2) by substituting 0 for r,

at $r=0$, $u = u_{max}$

$$u_{max} = \frac{\tau_w r_w^2}{2 r_w \mu}$$

$$\boxed{u_{max} = \frac{\tau_w r_w}{2\mu}} \quad \text{——— (3)}$$

Dividing equ(2) by (3), we get

$$\frac{u}{u_{max}} = \frac{\frac{\tau_w}{2 r_w \mu}(r_w^2 - r^2)}{\frac{\tau_w r_w}{2\mu}}$$

$$\frac{u}{u_{max}} = \frac{r_w^2 - r^2}{r_w^2}$$

$$\frac{u}{u_{max}} = 1 - \left(\frac{r}{r_w}\right)^2$$

Average velocity

$$u = \frac{\dot{m}}{\rho S}$$

surface Area,

$$S = \pi r_w^2$$

Elemental crosssectio

$$dS = 2\pi r\,dr$$

We know that,

$$\bar{v} = \frac{1}{s}\int_s u\,ds \Rightarrow u = \frac{\dot{m}}{\rho s} = \text{average velocity}$$

$$\boxed{\text{surface area } s = \pi r_w^2}$$

$$u = \frac{\tau_w}{2\mu r_w}\left(r_w^2 - r^2\right)$$

$$\boxed{\text{Elemental cross section } ds = 2\pi r\,dr}$$

$$v = \frac{1}{s}\int_s \frac{\tau_w}{2\mu r_w}\left(r_w^2 - r^2\right) ds$$

$$v = \frac{\tau_w}{\mu}\int_0^{r_w} \frac{r_w^2 - r^2}{r_w^3}\, r\,dr = \frac{\tau_w}{\mu r_w^3}\left[r_w^2\frac{r_w^2}{2} - \frac{r^4}{4}\right]$$

$$v = \frac{\tau_w}{\mu r_w^3}\left[\frac{r_w^4}{2} - \frac{r^4}{4}\right]$$

$$\bar{V} = \frac{\tau_w}{\mu r_w^3}\int_0^{r}\left[\frac{r_w^4}{2} - \frac{r^4}{4}\right]$$

$$\boxed{u_{max} = \frac{\tau_w r_w}{2\mu}}$$

$$V = \frac{\tau_w r_w^4}{4\mu r_w^3}$$

$$\boxed{\bar{V} = \frac{\tau_w r_w}{4\mu}} \quad \text{———} (4)$$

$$\frac{\bar{V}}{U_{max}} = \frac{\left(\tau_w r_w / 4\mu\right)}{\left(\tau_w r_w / 2\mu\right)} \Rightarrow \frac{\bar{V}}{U_{max}} = \frac{1}{2}$$

(or)

$$\boxed{\frac{\bar{V}}{U_{max}} = 0.5} \quad \text{———} (5)$$

* Hagen-poiseuille equation:-

Hagen poiseuille equation is used to estimate the viscosity of the fluid.

$$\bar{V} = \frac{\tau_w r_w}{4\mu}$$

$$= \frac{\Delta P}{L} \frac{r_w^2}{8\mu}$$

$$\bar{V} = \frac{\Delta P}{L} \frac{D^2}{32\mu}$$

$$\boxed{\frac{\Delta P}{L} = \frac{32\mu\bar{V}}{D^2}} \quad \text{——— (1)}$$

$$\because \frac{dP}{dL} + \frac{2\tau_w}{r_w} = 0$$

$$\tau_w = -\frac{dP}{dL}$$

$$\frac{dP}{dL} = -\frac{2\tau_w}{r_w}$$

$$\frac{2\tau_w}{r_w} = \frac{dP}{dL}$$

$$\tau = \frac{dP\, r_w}{2}$$

This is known as Hagen-poiseuille equation from fanning friction factor

$$\boxed{\frac{\Delta P}{L} = \frac{2f\rho\bar{V}^2}{D}} \quad \text{——— (2)}$$

Equating equ(1) & (2) we get

$$\frac{32\mu\bar{V}}{D^2} = \frac{2f\rho\bar{V}^2}{D}$$

$$\frac{32\mu\bar{V}D}{2D^2\rho\bar{V}^2} = f$$

$$\boxed{f = \frac{16\mu}{D\bar{V}\rho}}$$

$$\boxed{f = \frac{16}{Re}} \quad \text{——— (3)}$$

Equ(3) is the Hagen-poiseuille equation.

This equation uses to measure viscosity by measuring the pressure drop and volumetric flow rate through a tube of known length and diameter.

Prob ①:-

water flow through an annulous the outer dia of the inner pipe is 2 cm & the inner dia of the outer pipe is 4 cm. If the flow rate is 100 cc/sec. Determine the nature of flow ($\rho = 980\ kg/m^3$, $\mu = 0.85\ cP$)

Sol:- Given data,

$$D_i = 2\,cm = 0.02\,m$$

$$D_o = 4\,cm = 0.04\,m$$

$$Q = 100\,cc/sec = 100 \times 10^{-6}\ m^3/sec$$

$$\rho = 980\ kg/m^3$$

$$\mu = 0.85\ cP = 0.85 \times 10^{-2}\ kg/m.sec$$

$$\bar{V} = \frac{Q}{A} = \frac{100 \times 10^{-6}}{\pi/4\,(D_o - D_i)^2} = \frac{100 \times 10^{-6}}{\frac{\pi}{4}(0.04 - 0.02)^2} = \frac{\cancel{1}038 \times 10^{-2}}{0.318}\ m/sec$$

$$R_e = \frac{(D_o - D_i)\,\bar{V}\rho}{\mu}$$

$$R_e = \frac{(0.04 - 0.02)\,[980] \times (0.318 \times 10^{-2})}{0.85 \times 10^{-3}}$$

$$R_e = 733.62$$

$$R_e \geq 2100$$

ie., Flow is laminar flow

Prob ②:-

A smooth pipe of dia 8cm & 800m long carries water at the rate of $0.48\ m^3/hr$ (Assume kinetic viscosity of water as 0.015 stokes)
(i) calculate the loss of head
(ii) centre line velocity (iii) pressure drop
(iv) velocity head & shear stress at 3cm from pipe wall

sol:- Given data,

Dia of smooth pipe, $D = 8cm = 0.08\ m$

$S = \pi/4\ D^2 = \frac{\pi}{4}(0.08) = 5.025 \times 10^{-3}$

length, $L = 800m$

shear stress, $3cm = 3 \times 10^{-2}\ m$

kinematic viscosity, $\nu = 0.015$ stokes $= 0.015 \times 10^{-2}\ m^2/sec$

$$q = 0.48\ m^3/hr = \frac{0.48}{60 \times 60}\ m^3/sec$$

$$q = 1.33 \times 10^{-4}\ m^3/sec$$

$$h_{fs} = \frac{\Delta P}{\rho}$$

$$\Delta P = \frac{32 \mu \bar{V} L}{D^2}$$ [From Hagen poiselli equation]

$\mu = \nu \rho$
$= 0.015 \times 10^{-2} \times 1000$
$= 0.15\ kg/msec$

$\bar{V} = q/s$

$$\bar{V} = \frac{1.33 \times 10^{-4}}{5.025 \times 10^{-4}}$$

$\bar{V} = 0.026\ m/sec$

$$\Delta P = \frac{32 \times 0.15 \times 0.026 \times 800}{(0.08)^2}$$

$\Delta P = 15600\ N/m^2$

$$h_f = \frac{\Delta P}{\rho} = \frac{15600}{1000}$$

$$\boxed{h_f = 15.6\ N\text{-}m/kg}$$

$$IN = \frac{kg/m^2 sec}{kg/m^3}$$

(ii) centre line viscosity

$$\frac{\bar{v}}{v_{max}} = 0.5$$

$$v_{max} = 0.026/0.5$$

$$v_{max} = 0.05 \text{ m/sec}$$

(iii) pressure drop, $\Delta P = 15600 \text{ N/m}^2$

(iv) $h_{fs} = 4 \frac{L}{D} \frac{\tau_w}{\rho}$

$$\tau_w = \frac{h_{fs} \times D \times \rho}{4D}$$

$$\tau_w = \frac{15.6 \times (3\times10^{-2})(1000)}{4 \times 800}$$

$$\tau_w = 0.146$$

$$\bar{v} = \frac{\tau_w r_w}{4\mu} = \frac{\tau_w D}{8\mu}$$

$$= \frac{0.146 \times 8\times10^{-2}}{8\times(0.15)} = 3.65\times10^{-3} \text{ m/sec}$$

* **Turbulent flow in pipes:-**

Turbulence is caused by the appearance of eddies that lead to lateral mixing.

In turbulent flow through a pipe, the velocity at the interface between fluid & the solid column wall is zero & there are no velocity components normal to the wall.

* within a thin volume immediately adjacent to the wall, the velocity gradient is essentially constant & the flow is viscous most of the time. This volume is

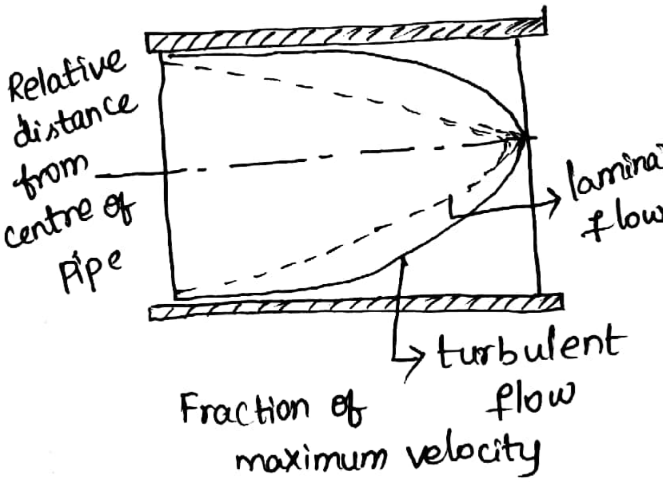
called "viscous sublayer" and eddies are moving into this layer from turbulent fluid.

* A "transition layer" (buffer layer) exists immediately adjacent to the viscous sublayer in which both viscous layer & shear due to eddy diffusion exists & is thin relatively.
* The bulk of the cross-section of the flowing stream is occupied by turbulent flow called the turbulent core. In the turbulent core, viscous shear is negligeble in comparision with that due to eddy viscosity $\rightarrow \mu_t$ (nothing but turbulent viscosity)

* velocity distribution in turbulent flow:-

Velocity distribution for a Newtonian fluid moving in laminar and turbulent flows are shown in fig.

* The curve for turbulent flow is much flatter than that of laminar flow, & the difference b/w the avg velocity & the maximum velocity is considerably less.



* At the centre line, the turbulence is isotropic (having same optical properties in all directions).
* It is customary to express the velocity distribution in turbulent flow not as velocity Vs distance but interms of dimensionless parameters.

Friction velocity $= u^*$

$$u^* = \bar{V}\sqrt{\frac{f}{2}} = \sqrt{\frac{\tau_w}{\rho}}$$

velocity co-efficient; dimensionless; $u^+ = \frac{u}{u^*}$

Distance, dimensionless; $y^+ = \frac{y u^* \rho}{\mu} = \frac{y}{\mu}\sqrt{\tau_w \rho}$

where,
y = Distance from wall of the tube

ε $r_w = r + y$

Equations relating u^+ to y^+ are called universal velocity distribution laws.

<u>Universal velocity distribution equations</u>

For viscous sub layer

$$\tau = \tau_w$$

$$\tau_w = -\mu \frac{du}{dr}$$

$$\tau_w = \mu \frac{du}{dy}$$

$$\int_0^u du = \frac{\tau_w}{\mu}\int_0^y dy$$

$$u = \frac{\tau_w}{\mu} y$$

$$u = \frac{\tau_w}{\rho}\frac{\rho y}{\mu}$$

$$u = (u^*)^2 \frac{\rho y}{\mu}$$

$$\frac{u}{u^*} = \frac{\rho u^* y}{\mu}$$

$$u^+ = y^+$$

$$\therefore \quad y = \frac{y u_* \rho}{\mu}$$

where,

$$u_* = \sqrt{\frac{\tau_w}{\rho}}$$

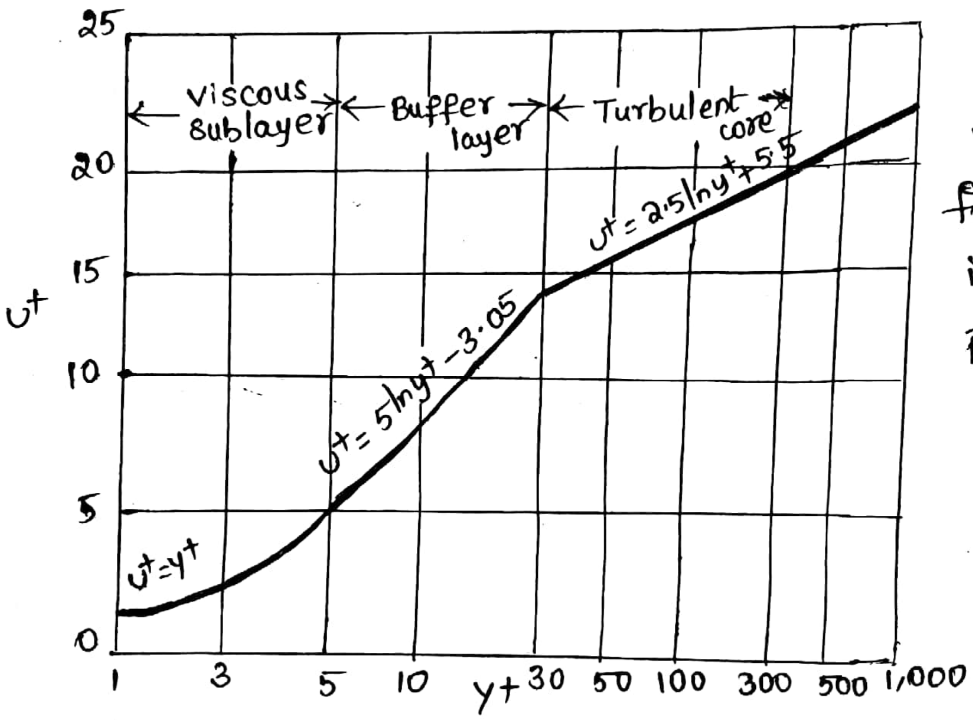
$$(u_*)^2 = \frac{\tau_w}{\rho}$$

$$\therefore u^+ = \frac{u}{u_*}$$

For laminar sub-layer ; $u^+ = y^+ \quad (0 < y^+ < 5)$

For buffer layer , $u^+ = 5.00 \ln y^+ - 3.05 \quad (5 < y^+ < 30)$

For turbulent core , $u^+ = 2.5 \ln y^+ + 5.5 \quad (y^+ > 30)$



velocity distribution for turbulent flow in a smooth pipe

* The flow quantities include average velocity, maximum velocity at the center of the pipe, τ_w, friction factor relation with average velocity , Reynolds number & KE correction factors α & β

$$u^+ = 2.5 \ln y^+ + 5.5 \quad \text{Eq}$$

$$u_c^+ = 2.5 \ln y_c^+ + 5.5 \rightarrow \text{at the centre line of pipe}$$

$$u_c^+ = \frac{u_{max}}{u^*}$$

$$y_c^+ = \frac{r_w u^*}{\nu}$$

$$\frac{\bar{V}}{u_{max}} = \frac{1}{1+3.75\sqrt{f/2}}$$

* **Reynolds Number friction factor law for smooth pipes:-**

$$y_c^+ = \frac{r_w}{\nu}\bar{V}\sqrt{f/2}$$

$$= \frac{D\bar{V}}{2\nu}\sqrt{f/2} = \frac{Re}{2}\sqrt{f/2}$$

$$\therefore \boxed{y_c^+ = Re\sqrt{f/8}}$$

$$\boxed{u_c^+ = \frac{1}{\sqrt{f/2}} + 3.75}$$

$$\boxed{u_c^+ = 2.5\ln y_c^+ + 5.5}$$

$$\frac{1}{\sqrt{f/2}} + 3.75 = 2.5\ln y_c^+ + 5.5$$

$$\frac{1}{\sqrt{f/2}} = 2.5\ln\left(Re\sqrt{f/8}\right) + 1.75$$ (von karmann equation)

* **Effect of Roughness:-**

It has been found that in turbulent flow, a rough pipe leads to a larger friction factor for a given Reynolds number than a smooth pipe.

If a rough pipe is smoothed, the friction factor is reduced. when further smoothing brings no further reduction in the friction factor for a given Reynolds number, the tube is said to be hydraulically smooth.

$$f = f\left(Re, K/D\right)$$

where

K = roughness parameter

* All new commercial pipes seem to have same type of roughness & that each material of construction has its own characteristic roughness parameter.
* Roughness has no appreciable effect on friction factor for laminar flow unless 'K' is so large that the measure of the diameter becomes uncertain.

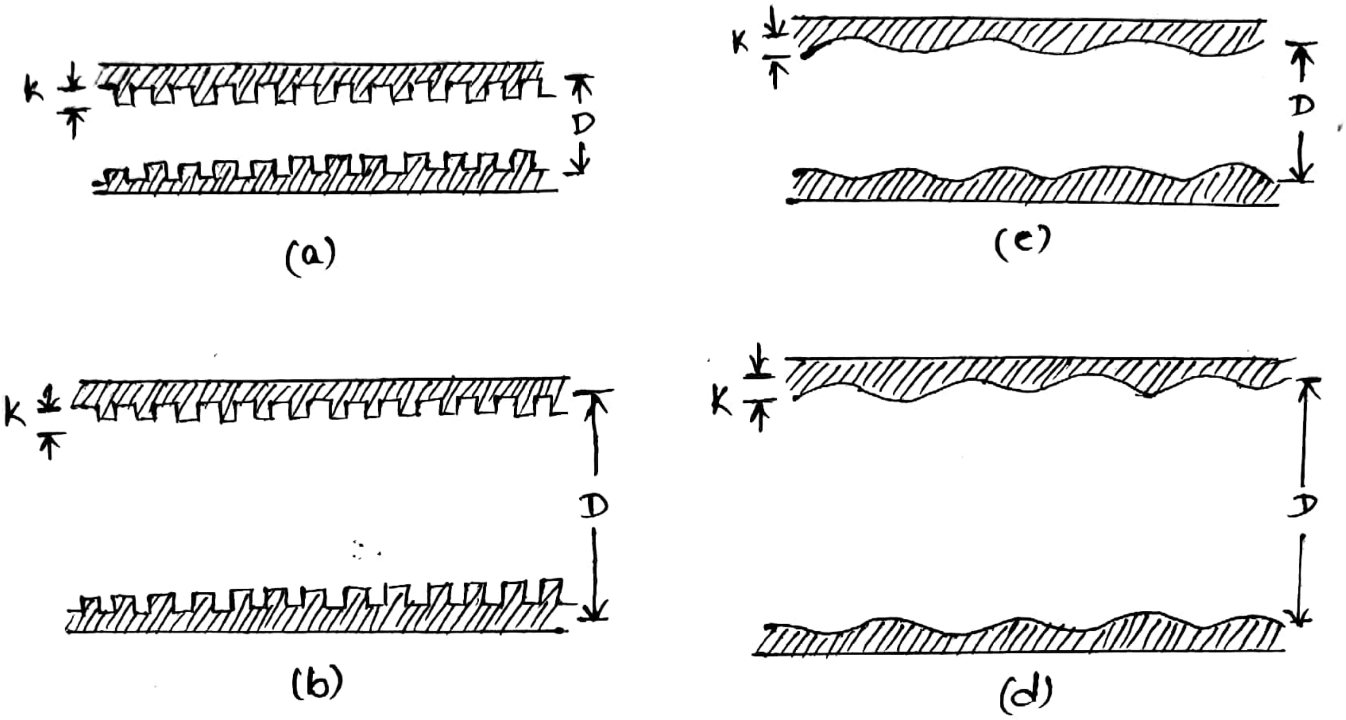
$$f = 0.046\, Re^{-0.2} \quad (\text{For } 50{,}000 < Re < 1\times10^6)$$

$$f = 0.0014 + \frac{0.125}{Re^{0.32}} \quad (\text{For } 3000 < Re < 3\times10^6)$$

'K' for wrought iron steel pipe $= 1.5\times10^{-4}$ m

* copper and brass pipes may be considered hydraulically smooth.
* The variation of friction factor with Reynolds number for various materials of construction and (K/d) values is shown in fig (a)
* The measurement of roughness parameter for various rough pipes is shown in fig(b) as

Types of roughness



* Friction factor in Turbulent flow :-

The parameter commonly used in the study of turbulent flow is the fanning friction factor (f).

It is defined as the ratio of the wall shear stress to the product of the density & the velocity head $\left(\frac{\bar{V}^2}{2}\right)$

$$f = \frac{\tau_w}{\rho \bar{V}^2/2} = \frac{2\tau_w}{\rho \bar{V}^2}$$

$$h_{fs} = \frac{2\tau_w}{\rho r_H} L = \frac{\Delta P_s}{\rho}$$

$$= \frac{f \bar{V}^2 L}{r_H}$$

$$= \frac{4fL \cdot \bar{V}^2}{2D}$$

$$\therefore f = \frac{\Delta P_s D}{2L\rho \bar{V}^2}$$

$$\therefore \boxed{\frac{\Delta P_s}{\rho} = \frac{2fL\bar{V}^2}{D}}$$

This equation is called Fanning equation. It is used to calculate the friction loss.

* Flow in Non-circular channels:-

In evaluating skin friction in channels of non-circular cross-section, the diameter in the Reynolds number must be the equivalent diameter (D_e)

$$f = \frac{\Delta P_s D_e}{2L \rho \bar{V}^2} \quad \text{and}$$

$$R_e = \frac{\rho \bar{V} D_e}{\mu}$$

where, $D_e = 4 r_H \rightarrow$ Hydraulic radius

Hydraulic radius is defined as the ratio of the cross-sectional area of the channel to the wetted perimeter of the channel.

$$r_H = \frac{S}{L_p} = \frac{\text{cross-sectional area of the channel}}{\text{wetted perimeter of the channel}}$$

For circular tube, $r_H = \dfrac{\pi D^2/4}{\pi D}$

$$= D/4$$

For annular space b/w two concentric pipes, the hydraulic radius is

$$r_H = \frac{\pi}{4} \frac{(D_o^2 - D_i^2)}{\pi (D_o + D_i)} = \frac{D_o - D_i}{4}$$

$$D_c = (D_o - D_i)$$

where,

D_i = inside diameter of the annulus

D_o = outside diameter of the annulus

Hydraulic radius of a duct with a width

$$b = \frac{b^2}{4b}$$

Equivalent diameter of a square duct with a width of side

$$b = 4\left(\frac{b^2}{4b}\right)$$

$$= b$$

For flow b/w $||^{el}$ plates,

when the distance b/w them 'H' is much smaller than the width of the plates (B), the equivalent diameter is

$$D_e = \frac{4BH}{2B} = 2H$$

For layer flow

$$D_e = \frac{4A}{L_p} = \frac{4w\delta}{w} = 4\delta$$

* **Reynolds Number & friction factor for Non-Newtonian fluids:-**

friction factor, $f = \dfrac{\Delta P_s D}{2L\rho\bar{V}^2}$ ——— (1)

Reynolds number for Newtonian fluid is

$$Re = \frac{DV\rho}{\mu}$$

Reynolds Number for Non-Newtonian fluid is

$$Re_n = 2^{3-n}\left(\frac{n'}{3n'+1}\right)^{n'} \frac{D^{n'}\rho\bar{V}^{2-n}}{K'}$$

friction factor for Non-Newtonian is

$$f = \frac{2^{n'+1} k'}{D^{n'} \rho \bar{V}^{2-n}} \left(3 + \frac{1}{n'}\right)^{n'}$$

From the above equation a Reynolds number, Re for Newtonian fluid can be defined, on the assumption that for laminar flow.

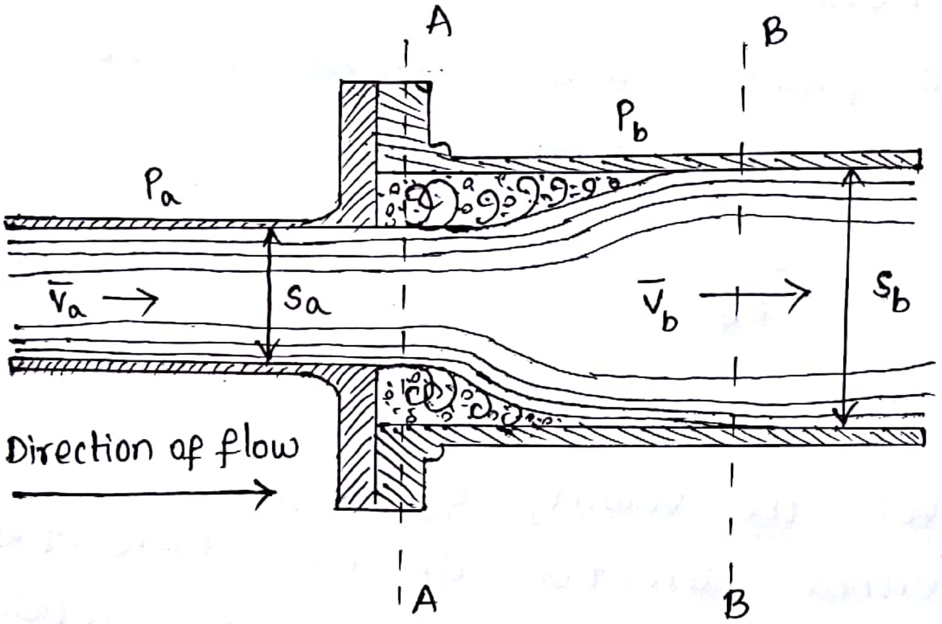
$$f = \frac{16}{Re}$$

* **Friction From changes in velocity or Direction :-**

whenever the velocity of a fluid is changed in either direction or magnitude, friction is generated in addition to the skin friction resulting from flow through the straight (tube) pipe. such friction includes form friction resulting from vertices that develop when normal streamlines are distributed and when boundary layer separation occurs. often these effects cannot be calculated precisely, & it is necessary to rely on emperical data.

(a) **Friction loss from sudden expansion of cross-section**

cross section of the pipe is suddenly enlarged, the fluid stream seperates from the wall & issues at a jet into the enlarged section. The jet then expands to fill the entire cross section of the larger conduit. The space between the expanding jet and the conduit wall is filled with fluid in vortex motion characteristic of boundary layer separation & considerable friction is generated within this space. This effect is shown in figure.



Flow at sudden enlargement of cross section

The friction loss (h_{fe}) from a sudden expansion of cross section is proportional to the velocity head of the fluid in the small conduit can be written as

$$h_{fe} = K_e \frac{\bar{V}_a^2}{2} \quad \text{--- (1)}$$

$K_e \rightarrow$ expansion loss co-efficient

$\bar{V}_a \rightarrow$ average velocity in smaller conduit or upstream

$\bar{V}_b \rightarrow$ average velocity in larger conduit or downstream

AA & BB $\rightarrow$ control volume defined by sections

→ Gravity force do not appear because of pipe is horizontal.

→ wall friction is negligible because the wall is relatively short.

→ No velocity gradient at the wall between the sections.

→ The only forces; these forces are pressure forces on Section AA & BB.

The momentum equations gives

$$p_a S_a - p_b S_b = \dot{m}(\beta_b \bar{V}_b - \beta_a \bar{V}_a) \quad \text{—— (2)}$$

Since $z_a = z_b$,

Bernoulli equation $\left(\frac{p_a}{\rho} + gz_a + \frac{\alpha_a \bar{V}_a^2}{2} = \frac{p_b}{\rho} + gz_b + \frac{\alpha_b \bar{V}_b^2}{2}\right)$

can be written for this situation as

$$\frac{p_a - p_b}{\rho} = \frac{\alpha_b \bar{V}_b^2 - \alpha_a \bar{V}_a^2}{2} + h_{fe} \quad \text{—— (3)}$$

For usual flow conditions, $\alpha_a = \alpha_b = 1$ & $\beta_a = \beta_b = 1$, & these correction factors are disregarded. Also elimination of $p_a - p_b$ between equ (2) & (3) yields,

Since $\frac{\dot{m}}{S_b} = \rho \bar{V}_b$

$$h_{fe} = \frac{\bar{V}_a - \bar{V}_b}{2} \quad \text{—— (4)}$$

From equs, $[\dot{m} = \rho_a \bar{V}_a S_a = \rho_b \bar{V}_b S_b = \rho \bar{V} S]$, $\bar{V}_b = \bar{V}_a (S_a / S_b)$

Since ρ is const & equ (4) can be written as

$$h_{fe} = \frac{\bar{V}_a^2}{2}\left(1 - \frac{S_a}{S_b}\right)^2 \quad \text{—— (5)}$$

Comparing equs (1) & (5) shows that

$$K_e = \left(1 - \frac{S_a}{S_b}\right)^2 \quad \text{—— (6)}$$

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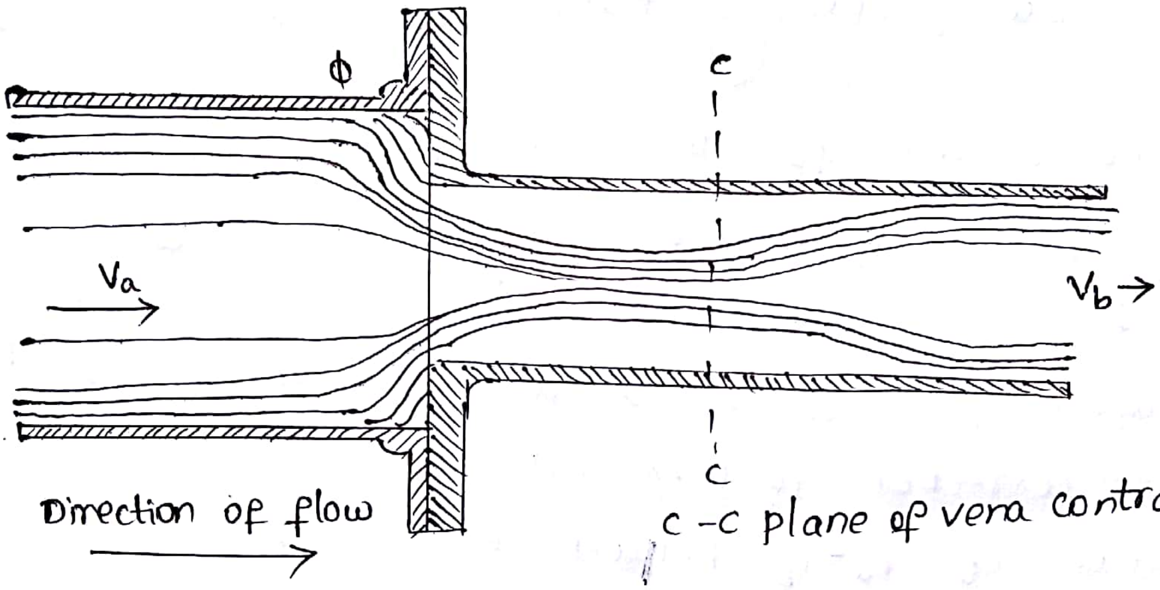
A correction for α & β should be made if the type of flow between sections differs. For example, if the flow is in larger pipe is laminar & that in the smaller pipe turbulent, α_b should be taken as 2 & β_b as 4/3 in equs 2 & 3.

* Friction loss from sudden contraction of cross section:-



Flow at sudden contraction of cross section

When the cross section of the conduit is suddenly reduced, the fluid stream cannot follow around the sharp corner and the stream breaks contact with the wall of the conduit.

The cross section of minimum area at which the jet changes from contraction to an expansion is called the "vena contracta". The flow pattern of a sudden contraction is shown in fig.

The section cc is drawn at vena contracta vortices appear as shown in figure.

The friction loss from sudden contraction is proportional to the velocity head in the smaller conduit and can be calculated by the equation

$$h_{fc} = K_c \frac{\bar{V}_b^2}{2} \quad \text{——— (7)}$$

$K_c \rightarrow$ contraction loss co-efficient

$V_b \rightarrow$ average velocity in the smaller or down stream

→ Experimentally for laminar flow, $K_c < 0.1$ and the contraction loss h_{fc} is negligible.

→ For turbulent flow,

K_c is given by the emperical equation

$$K_c = 0.4\left(1 - \frac{S_b}{S_a}\right)$$

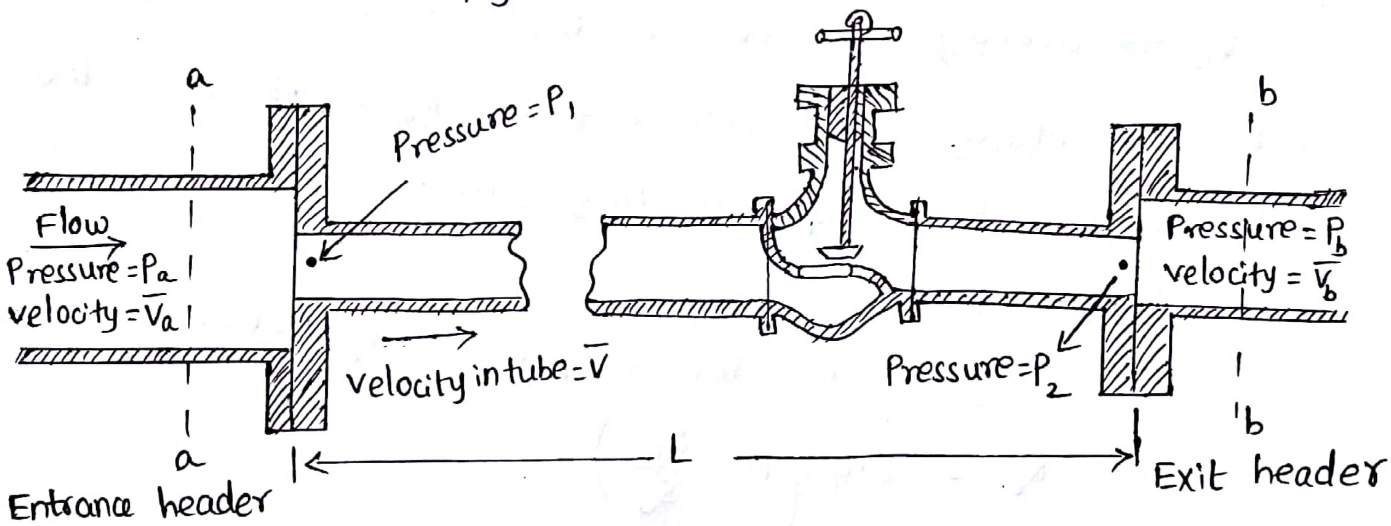
where S_a & S_b are the cross sectional areas of upstream & downstream conduits, respectively.

* Form friction losses in Bernoulli equation :-

Form friction losses are incorporated in the h_f term of

$$\frac{P_a}{\rho} + gZ_a + \frac{\alpha_a \bar{V}_a^2}{2} + \eta w_p = \frac{P_b}{\rho} + gZ_b + \frac{\alpha_b \bar{V}_b^2}{2} + h_f \quad \text{—(1)}$$

They are combined with the skin friction losses of the straight pipe to give the total friction loss. consider, for example, the flow of incompressible fluid through two enlarged headers, the connecting tube and open globe valve shown in figure.



Flow of Incompressible fluid through typical assembly

Let v be the average velocity in the tube, D the diameter of the tube and L the length of the tube. The skin friction loss in the straight tube is by $4f(L/D)(\bar{V}^2/2)$; the contraction loss at the entrance to the tube by $K_c(\bar{V}^2/2)$; the expansion loss at the exit of the tube is $K_e(\bar{V}^2/2)$; and the friction loss in the globe valve is $K_f(\bar{V}^2/2)$. When skin friction in the entrance and exit headers is neglected, the total friction is,

$$h_f = \left(4f\frac{L}{D} + K_c + K_e + K_f\right)\frac{\bar{V}^2}{2} \quad \text{——— (2)}$$

To write the Bernoulli equation for this assembly, take station a in the inlet header and station b in the outlet header. Because there is no pump between stations a and b, $W_p = 0$; also α_a and α_b can be taken as 1.0, the kinetic energy term can be canceled, and equ(1) becomes

$$\frac{P_a - P_b}{\rho} + g(Z_a - Z_b) = \left(4f\frac{L}{D} + K_c + K_e + K_f\right)\frac{\bar{V}^2}{2} \quad \text{——— (3)}$$

Problem :-

Crude oil having a specific gravity of 0.93 and a viscosity of 4 cP is draining by gravity from the bottom of a tank. The depth of liquid above the drawoff connection in the tank is 6m. The line from the drawoff is 3-in. schedule 40 pipe. Its length is 45m, and it contains one ell and two gate valves. The oil discharges into the atmosphere 9m below the drawoff connection of the tank. What flowrate, in cubic meters per hour, can be expected through the line?

Sol:- The quantites needed are

$\mu = 0.004\ kg/m.s \qquad L = 45m$

$D = \frac{3.068}{12} = 0.256\ ft\ (APP.3) = 0.078\ m$

$\rho = 0.93 \times 998 = 928\ kg/m^3$

For fittings, from k_f ($90° = 0.75$)

$$\Sigma k_f = 0.75 + 2 \times 0.17 = 1.09$$

From eq, $\dfrac{P_a}{\rho} + gz_a + \dfrac{\alpha_a \bar{V}_a^2}{2} = \dfrac{P_b}{\rho} + gz_b + \dfrac{\alpha_b \bar{V}_b^2}{2} + h_f$,

assuming $\alpha_b = 1$ and since $P_a = P_b$ and $\bar{V}_a = 0$,

$$\frac{\bar{V}_b^2}{2} + h_f = g(z_a - z_b) = 9.80665(6+9) = 147.1\, m^2/s^2 \quad \text{— (1)}$$

Use equ, $h_f \left(4f\frac{L}{D} + k_c + k_e + k_f\right)\dfrac{\bar{V}^2}{2}$. There is no final expansion loss, since the kinetic energy of the stream is already accounted for and $k_e = 0$. From equ, $k_c = 0.4\left(1 - \dfrac{S_b}{S_a}\right)$, since S_a is very large, $k_c = 0.4$

Hence,

$$h_f = \left(4f\frac{L}{D} + k_c + \Sigma k_f\right)\frac{\bar{V}_b^2}{2}$$

$$= \left(\frac{4 \times 45 f}{0.078} + 0.4 + 1.09\right)\frac{\bar{V}_b^2}{2} = (2{,}308f + 1.49)\frac{\bar{V}_b^2}{2}$$

From equ (1),

$$\frac{V_b^2}{2} + h_f = \frac{\bar{V}_b^2}{2}(1 + 2{,}308f + 1.49) = 147.1$$

$$V_b^2 = \frac{147.1 \times 2}{2.49 + 2308f} = \frac{294.2}{2.49 + 2308f}$$

use friction factor plot for circular pipes to find f.

For this problem,

$$R_e = \frac{0.078 \times 928 \bar{V}_b}{0.004} = 18{,}096\, \bar{V}_b$$

$$\frac{k}{D} = 0.00015 \times \frac{0.3048}{0.078}$$

$$= 0.00059$$

Trials give the following;

$\bar{V}_{b,est.}$, m/s	$R_e \times 10^{-4}$	f (from above figure)	$\bar{V}_{b,cal}$; m/s
4.00	7.23	0.0056	4.37
4.37	7.91	0.0055	4.40
4.40	7.96	0.0055	4.40

The cross-sectional area of the pipe is 0.0513 ft^2, or 0.00477 m^2 (APP. 3), and the flow rate is $4.40 \times 3600 \times 0.00477 = 75.6\ m^3/h$.